1

Solve the following integer programming problem using branch-and-bound technique.

Maximize $Z = 10x_1 + 20x_2$, subject to

$6x_1 + 8x_2 \le 48$; $x_1 + 3x_2 \le 12$

$x_1, x_2 \ge 0$ and integers

Soln

Let $6x_1 + 8x_2 = 48$

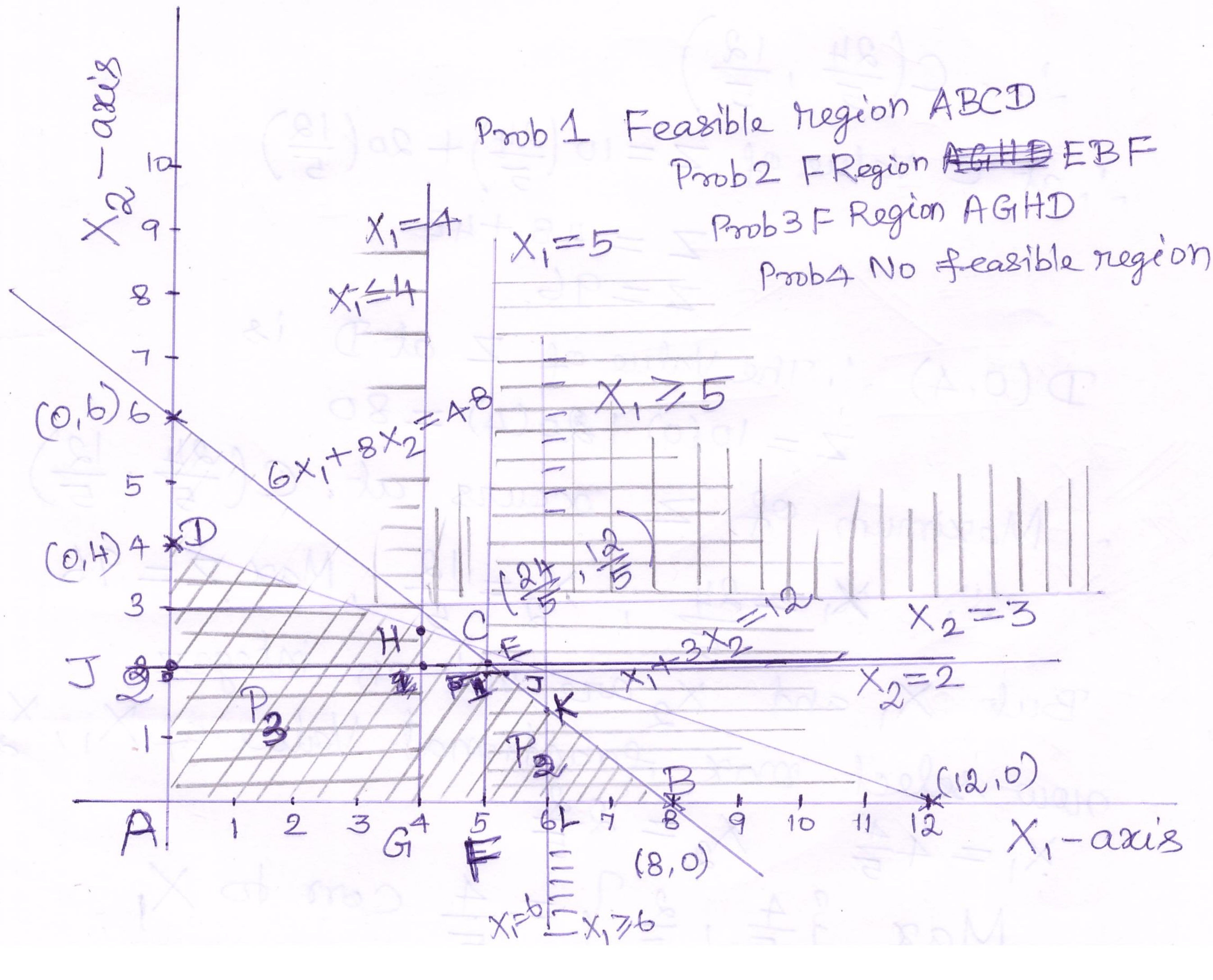
Put $x_2 = 0$ $\therefore$ $6x_1 = 48$ $x_1 = 8$ $\therefore$ $(8, 0)$

Put $x_1 = 0$ $\therefore$ $8x_2 = 48$ $x_2 = 6$ $(0, 6)$

Let $x_1 + 3x_2 = 12$

Put $x_2 = 0$ $\therefore$ $x_1 = 12$ $(12, 0)$

when $x_1 = 0$ $3x_2 = 12$ $x_2 = 4$, $(0, 4)$



Max $Z = 10X_1 + 20X_2$

$A(0,0)$ at the point A, $Z = 0$

$B(8,0)$ at the point B $Z = 10(8) = 80$

C be the point of intersection of the straight lines

$6X_1 + 8X_2 = 48$ —— (1) and $X_1 + 3X_2 = 12$ —— (2)

Solving (1) & (2)

(1) × 1 $6X_1 + 8X_2 = 48$

(2) × 6 $6X_1 + 18X_2 = 72$

$-10X_2 = -\cancel{24}$ $\quad X_2 = \frac{\cancel{24}}{10} = \frac{12}{5}$

$X_2 = \frac{12}{5}$ sub in (2) $X_1 + 3\left(\frac{12}{5}\right) = 12$

$X_1 = 12 - \frac{36}{5}$

$X_1 = \frac{60-36}{5} = \frac{24}{5}$

$\therefore C\left(\frac{24}{5}, \frac{12}{5}\right)$

$\therefore$ at C Value of $Z = 10\left(\frac{24}{5}\right) + 20\left(\frac{12}{5}\right)$

$Z = 48 + 48$

$Z = 96$

$D(0,4)$ $\therefore$ The Value of Z at D is

$Z = 10(0) + 20(4) = 80$

$\therefore$ Maximum of Z occurs at $C\left(\frac{24}{5}, \frac{12}{5}\right)$

$\therefore X_1 = \frac{24}{5}$, $X_2 = \frac{12}{5}$, Max $Z = 96$

But X_1 and X_2 are not an integer

now select max fractional Value of X_1, X_2

$X_1 = 4\frac{4}{5}$ $\quad X_2 = 2\frac{2}{5}$

Max $\left\{\frac{4}{5}, \frac{2}{5}\right\} = \frac{4}{5}$ corr to X_1

max fractional part occurs at X_1

$X_1 = \frac{24}{5} = 4\frac{4}{5}$ integer part of $X_1 = 4$

$Z_U = 96$

$\therefore$ Problem 1

$X_1 \geq 5$ → Problem 2 $X_1 \leq 4$ → Problem 3

$Z_L = 80$

Problem 2
Feasible region EBF

Problem 3
FR AGHD

$(X_1 = 4, X_2 = 2)$

~~For Prot~~ For Problem 2.

E (5,0), the Value of $Z = 10(5) + 0 = 50$

B (8,0) the Value of $Z = 10(8) + 0 = 80$

F be the point of intersection of the st lines

$X_1 = 5$ and $6X_1 + 8X_2 = 48$

$\therefore \quad 6(5) + 8X_2 = 48$

$8X_2 = 18$

$X_2 = \frac{18}{8} = \frac{9}{4}$

$\therefore$ ~~E~~ $F\left(5, \frac{9}{4}\right)$

The Value of Z at F is $Z = 10(5) + 20\left(\frac{9}{4}\right)$

$= 50 + 45 = 95$

$\therefore$ Max Value occur at $F\left(5, \frac{9}{4}\right)$

$\therefore$ optimal solution of problem 2 is

$X_1 = 5 \quad X_2 = \frac{9}{4}, \quad z = 95 > Z_L$

$\therefore$ go to next step.

$\therefore$ Here X_2 is not an integer.

$X_2 = 2\frac{1}{4}$

Problem 2

$X_2 \geq 3$ → Problem 4 $X_2 \leq 2$ → Problem 5

For problem 4 there is No feasible region. (4)

For problem 5 feasible region ~~FBI~~J $Z = 10X_1 + 20X_2$

$F(5,0)$ $Z = 10(5) + 0 = 50$

$B(8,0)$ $Z = 10(8) + 0 = 80$

$I(5,2)$ $Z = 10(5) + 20(2) = 90$

J be the point of intersection of $X_2 = 2$ and

$6X_1 + 8X_2 = 48$

$\therefore X_2 = 2 \Rightarrow 6X_1 + 16 = 48$

$6X_1 = 32$

$X_1 = \frac{32}{6} = \frac{16}{3}$

$\therefore J\left(\frac{16}{3}, 2\right)$ at J Value of $Z = 10\left(\frac{16}{3}\right) + 20(2)$

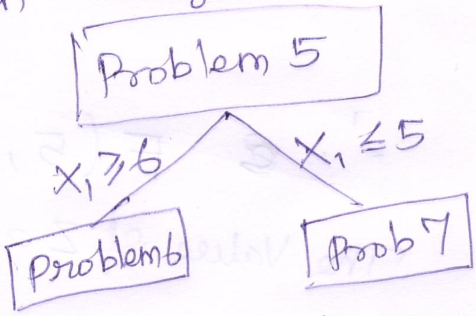
$Z = 93.33 \geq Z_L$

$\therefore$ optimal soln of problem 5 is

$X_1 = \frac{16}{3}$ $X_2 = 2.$

Here X_1 is not an integer

$X_1 = 5\frac{1}{3}$



Problem 6

$\therefore$ Feasible region of Problem 6 is LBK

$L(6,0)$ at L Value of ~~max~~ $Z = 10(6) = 60$

$B(8,0)$ at B Value of $Z = 10(8) = 80$

K be the point of intersection of $X_1 = 6$ and $6X_1 + 8X_2 = 48$

when $X_1 = 6$, $36 + 8X_2 = 48$

~~$8X_2 = \frac{48}{36} = \frac{8}{6} = \frac{4}{3}$~~

$\therefore K\left(6, \frac{3}{2}\right)$ $8X_2 = 12$

$X_2 = \frac{12}{8} = \frac{3}{2} = 1.5$

at $K(6, \frac{3}{2})$ the value of $Z = 10(6) + 20\left(\frac{3}{2}\right)$

$= 60 + 30 = 90$

$\therefore$ optimal solution for problem 6 is

$X_1 = 6 \quad X_2 = \frac{3}{2}$

Still X_2 is not an integer